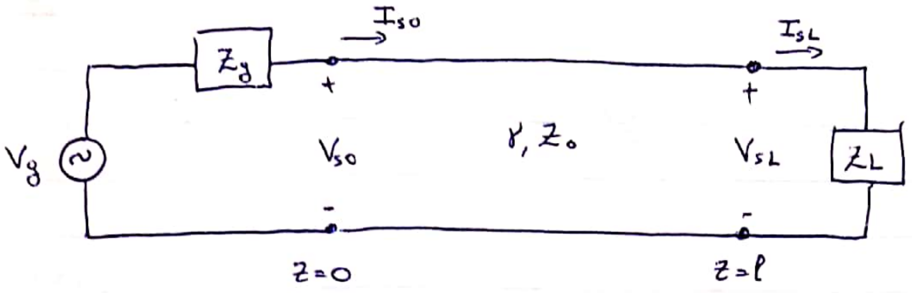
Circuito de Linha de Transmissão

Gerador - L.T. - Carga



Das eq. de L.T:

$$V_s(z) = V_{s0}^+ e^{-\gamma z} + V_{s0}^- e^{\gamma z} \quad (1)$$

$$I_s(z) = \frac{1}{Z_0}\left[V_{s0}^+ e^{-\gamma z} - V_{s0}^- e^{\gamma z} \right] \quad (2)$$

$I_s(z) = I_{s0}^+ e^{-\gamma z} + I_{s0}^- e^{\gamma z}$

Mas $Z_0 = -\frac{V_0^-}{I_0^-}$

A tensão na carga em $z = \ell$ é:

$$V_s(\ell) = V_{s0}^+ e^{-\gamma \ell} + V_{s0}^- e^{\gamma \ell} = V_{sL} \quad (3)$$

$$I_s(\ell) = \frac{1}{Z_0}\left[V_{s0}^+ e^{-\gamma \ell} - V_{s0}^- e^{\gamma \ell} \right] = I_{sL} \quad (4)$$

trabalhando as duas equações acima:

$$V_{s0}^+ = \frac{1}{2} e^{\gamma \ell} \left(V_{sL} + I_{sL} Z_0 \right) \quad (5)$$

$$V_{s0}^- = \frac{1}{2} e^{-\gamma \ell} \left(V_{sL} - I_{sL} Z_0 \right) \quad (6)$$

Se as impedâncias de cada lado da linha são dissimilares (linha/carga) $\Rightarrow$ $Z_L \neq Z_0$ haverá reflexão.

O coeficiente de reflexão em qualquer posição ao longo da linha é definido como a razão da onda de tensão refletida p/ a transmitida.

$$\Gamma(z) = \frac{V_{s0}^{-} e^{\gamma z}}{V_{s0}^{+} e^{-\gamma z}} = \frac{V_{s0}^{-}}{V_{s0}^{+}} e^{2\gamma z} \quad (7)$$

Inserindo as expressões p/ os coef. de tensão em termos da tensão na carga e corrente na carga, tem-se:

$$\Gamma(z) = \frac{V_{sL} - I_{sL} Z_0}{V_{sL} + I_{sL} Z_0} e^{-2\gamma l} e^{2\gamma z}$$

$$= \frac{\frac{V_{sL}}{I_{sL}} - Z_0}{\frac{V_{sL}}{I_{sL}} + Z_0} \cdot e^{2\gamma(z-l)}$$

$$\boxed{\Gamma(z) = \frac{Z_L - Z_0}{Z_L + Z_0} e^{2\gamma(z-l)}} \quad (8)$$

Assim, na carga em $z = l$:

$$\Gamma(l) = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Em função da posição:

$$\boxed{\Gamma(z) = \Gamma_L e^{2\gamma(z-l)}} \quad (9)$$

Expressando as eq. de L.T. em termos de Γ.

$$V_s(z) = V_{s0}^+ e^{-\gamma z} + V_{s0}^- e^{\gamma z} = V_{s0}^+ e^{-\gamma z}\left[1 + \frac{V_{s0}^- e^{\gamma z}}{V_{s0}^+ e^{-\gamma z}}\right]$$

$$I_s(z) = \frac{1}{Z_0}\left[V_{s0}^+ e^{-\gamma z} - V_{s0}^- e^{\gamma z}\right] = \frac{V_{s0}^+}{Z_0} e^{-\gamma z}\left[1 - \frac{V_{s0}^- e^{\gamma z}}{V_{s0}^+ e^{-\gamma z}}\right]$$

Assim:

$$\begin{cases} V_s(z) = V_{s0}^+ e^{-\gamma z}\left[1 + \Gamma(z)\right] & (10) \\ I_s(z) = \dfrac{V_{s0}^+}{Z_0} e^{-\gamma z}\left[1 - \Gamma(z)\right] & (11) \end{cases}$$

Calculando a Impedância de Entrada da L.T.:

$$Z_{in}(z) = \frac{V_s(z)}{I_s(z)} = Z_0 \frac{V_{s0}^+ e^{-\gamma z} + V_{s0}^- e^{\gamma z}}{V_{s0}^+ e^{-\gamma z} - V_{s0}^- e^{\gamma z}} \quad (12)$$

Usando (5) e (6):

$$V_{s0}^+ = \frac{1}{2} e^{\gamma l}\left(V_{sL} + I_{sL} Z_0\right)$$

$$= \frac{1}{2} e^{\gamma l} I_{sL}\left(V_{sL}/I_{sL} + Z_0\right)$$

$$V_{s0}^+ = \frac{I_{sL}}{2} e^{\gamma l}\left(Z_L + Z_0\right) \quad (13a)$$

da mesma forma:

$$V_{s0}^- = \frac{1}{2} e^{-\gamma l}\left(V_{sL} - I_{sL} Z_0\right) = \frac{I_{sL}}{2} e^{-\gamma l}\left(Z_L - Z_0\right) \quad (13b)$$

Inserindo 13a e 13b em 12 :

$$Z_{in}(z) = Z_0 \cdot \frac{e^{\gamma(\ell-z)}(Z_L + Z_0) + e^{-\gamma(\ell-z)}(Z_L - Z_0)}{e^{\gamma(\ell-z)}(Z_L + Z_0) - e^{-\gamma(\ell-z)}(Z_L - Z_0)}$$

$$= Z_0 \cdot \frac{Z_L\left[e^{\gamma(\ell-z)} + e^{-\gamma(\ell-z)}\right] + Z_0\left[e^{\gamma(\ell-z)} - e^{-\gamma(\ell-z)}\right]}{Z_L\left[e^{\gamma(\ell-z)} - e^{-\gamma(\ell-z)}\right] + Z_0\left[e^{\gamma(\ell-z)} + e^{-\gamma(\ell-z)}\right]}$$

Sabendo que:

$$\operatorname{senh}(\gamma\ell) = \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2} \qquad \cosh(\gamma\ell) = \frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2}$$

$$\tanh(\gamma\ell) = \frac{\sinh(\gamma\ell)}{\cosh(\gamma\ell)}$$

Logo:

$$Z_{in}(z) = Z_0 \cdot \frac{Z_L \cosh[\gamma(\ell-z)] + Z_0 \sinh[\gamma(\ell-z)]}{Z_0 \cosh[\gamma(\ell-z)] + Z_L \sinh[\gamma(\ell-z)]} \qquad (14)$$

dividindo num. e den. por $\cosh[\gamma(\ell-z)]$:

$$\boxed{Z_{in}(z) = Z_0 \cdot \frac{Z_L + Z_0 \tanh[\gamma(\ell-z)]}{Z_0 + Z_L \tanh[\gamma(\ell-z)]}} \qquad (15)$$

p/ uma linha sem perdas:

$\gamma = j\beta$ e Z_0 é puramente real:

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$$\tanh[\gamma(\ell-z)] = \tanh[j\beta(\ell-z)] = j\tan[\beta(l-z)]$$

Assim:

$$\boxed{Z_{in}(z) = Z_0 \cdot \frac{Z_L + jZ_0\tan[\beta(\ell-z)]}{Z_0 + jZ_L\tan[\beta(\ell-z)]}}$$

L.T. sem perdas (16)

Casos especiais:

1) linha sem perda em aberto:

$|Z_L| \to \infty \qquad \Gamma_L = 1$

$$\lim_{|Z_L|\to\infty}\{Z_{in}(z)\} = Z_0 . \lim_{|Z_L|\to\infty}\left\{\frac{Z_L + jZ_0\tan[\beta(\ell-z)]}{Z_0 + jZ_L\tan[\beta(l-z)]}\right\}$$

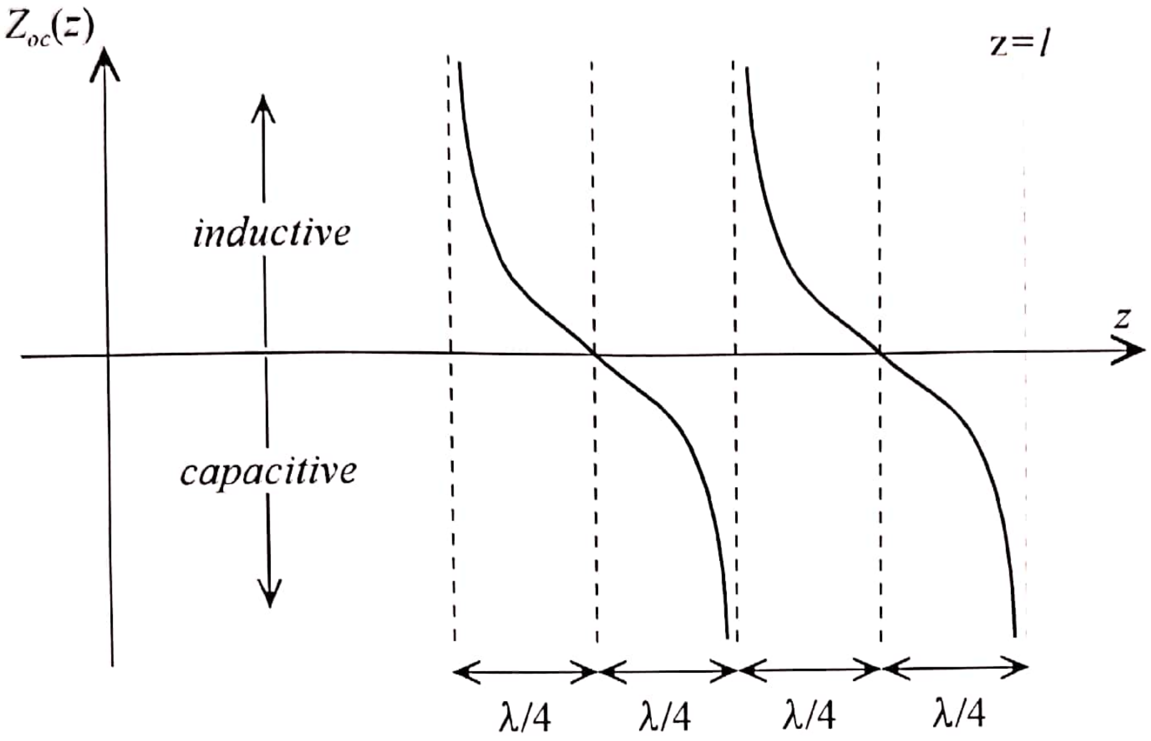
$$= Z_0 \frac{1}{j\tan[\beta(\ell-z)]} = -jZ_0\cot[\beta(\ell-z)] = Z_{oc}(z)$$

2) linha sem perda curto-circuitada

$|Z_L| \to 0 \ , \quad \Gamma_L \to -1$

$$\lim_{|Z_L|\to 0}\{Z_{in}(z)\} = Z_0 \lim_{|Z_L|\to 0}\left\{\frac{Z_L + jZ_0\tan[\beta(\ell-z)]}{Z_0 + jZ_L\tan[\beta(\ell-z)]}\right\}$$

$$= jZ_0\tan[\beta(\ell-z)] = Z_{sc}(z)$$


$Z_{oc}(z)$
z=l
inductive
capacitive
z
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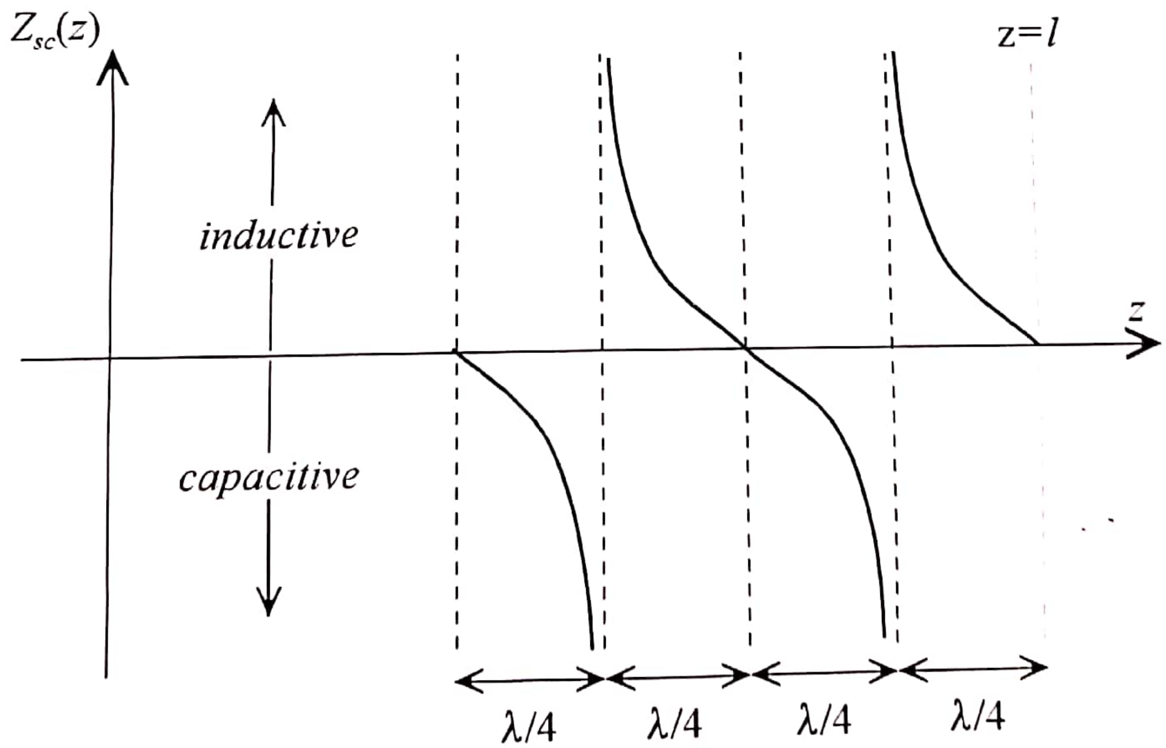
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λ/4
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